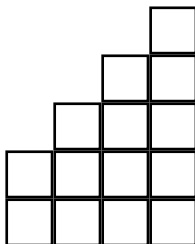


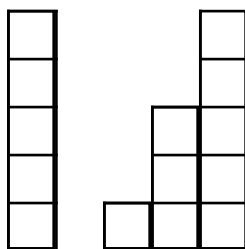
## Staircases

Here is an example of a kind of arrangement of square tiles that we'll call a *staircase*. It has 4 steps and the first step is of height 2.



**Definition:** For this lesson, we will define a staircase as a sequence of stacks of tiles in which each stack is one tile higher than the previous stack. *There must be two or more steps in the staircase and the first step can be of any height.*

These are *not* staircases:



- How many tiles would you need to build each of these staircases:
  - First step: 6. Number of steps: 7
  - First step: 7. Number of steps: 6
- There are two different 9-tile staircases:  $2 + 3 + 4$  and  $4 + 5$ .
  - Find three different 15-tile staircases.
  - Find four different 105-tile staircases.
- Exploration.** Find every possible staircase with each number of tiles from 1 to 34. Hints:
  - Work with other students.
  - Keep organized records of your work.
  - It is not necessary to draw the staircases.
  - Look for strategies: what numbers can be made into two-step staircases? three-step?
  - Look for patterns: what numbers are easiest? what numbers are impossible?
  - What is the average (mean) of a string of consecutive numbers?
  - How is the average related to the sum?
  - What is a good shortcut for adding consecutive numbers?

In general, how can you predict the number of ways a number can be written as a sum of consecutive numbers?

## Egyptian Fractions Challenge

Write each fraction as a sum of three or fewer unit fractions (fractions whose numerator is 1). One has been done for you. You don't have to do them in order. Don't use negative numbers.

$$\frac{4}{3} =$$

$$\frac{4}{4} =$$

$$\frac{4}{5} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$$

$$\frac{4}{6} =$$

$$\frac{4}{7} =$$

$$\frac{4}{8} =$$

$$\frac{4}{9} =$$

$$\frac{4}{10} =$$

$$\frac{4}{11} =$$

$$\frac{4}{12} =$$

$$\frac{4}{13} =$$

$$\frac{4}{14} =$$

$$\frac{4}{15} =$$

$$\frac{4}{16} =$$

$$\frac{4}{17} =$$

$$\frac{4}{18} =$$

$$\frac{4}{19} =$$

$$\frac{4}{20} =$$

$$\frac{4}{21} =$$

$$\frac{4}{22} =$$

$$\frac{4}{23} =$$

$$\frac{4}{24} =$$

$$\frac{4}{25} =$$

$$\frac{4}{26} =$$

$$\frac{4}{27} =$$

$$\frac{4}{28} =$$

$$\frac{4}{29} =$$

$$\frac{4}{30} =$$

$$\frac{4}{31} =$$

$$\frac{4}{32} =$$

$$\frac{4}{33} =$$

$$\frac{4}{34} =$$

$$\frac{4}{35} =$$

$$\frac{4}{36} =$$

$$\frac{4}{37} =$$

$$\frac{4}{38} =$$

$$\frac{4}{39} =$$

$$\frac{4}{40} =$$

$$\frac{4}{41} =$$

$$\frac{4}{42} =$$

$$\frac{4}{43} =$$

$$\frac{4}{44} =$$

$$\frac{4}{45} =$$

$$\frac{4}{46} =$$

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$$\frac{4}{56} =$$