Grades 7–10

ALGEBRA LAB GEAR

Algebra 1

- Polynomial Arithmetic
- Equations and Identities
- Quadratics
- Factoring
- Graphing Connections

A1

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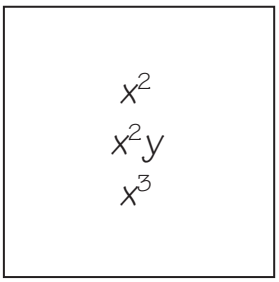
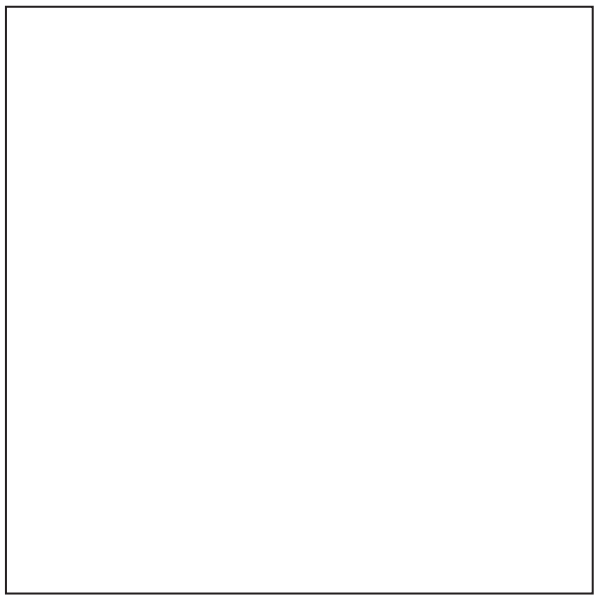
Face to Face

Each block has six faces. The faces are all rectangles (or squares). There are ten different-sized faces as shown below.

- Beneath each rectangular face shown, write its dimensions.
- Inside each rectangular face, list *all the blocks* that fit that face perfectly. The first one is done for you.

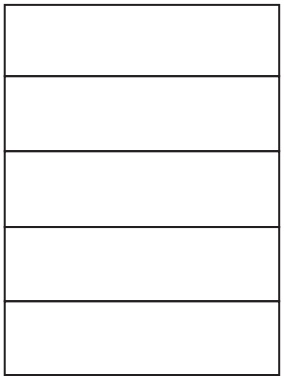


1. x by x

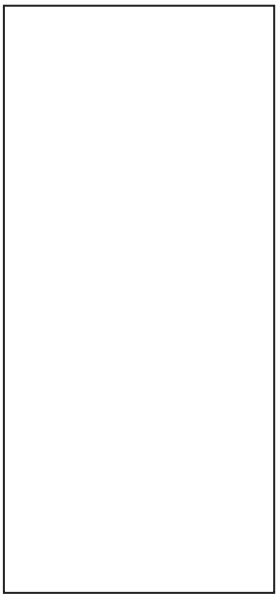


2. _____ by _____

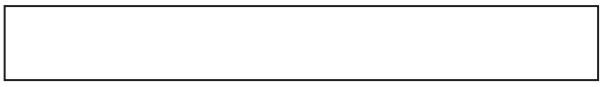
3. _____ by _____



4. _____ by _____



6. _____ by _____



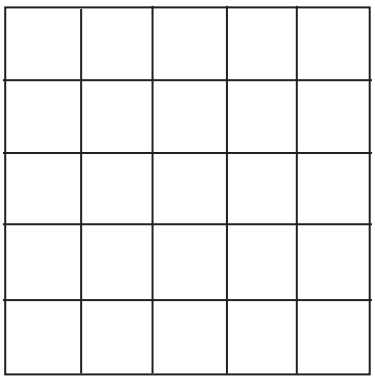
5. _____ by _____

7. _____ by _____

List the blocks here:

8. _____ by _____

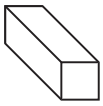
9. _____ by _____

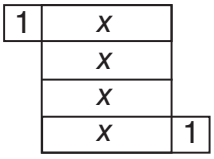


10. _____ by _____

Surface Area

1. Take out one of each of the different Lab Gear blocks. Write the surface area of the block. Combine like terms to make your answer compact. Record your answer in the chart below. (The first one has been done as an example.)

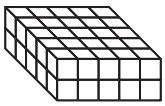




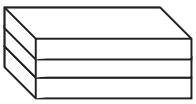
Lab Gear Block	Surface Area
x	$x + x + x + x + 1 + 1 = 4x + 2$
$5x$	
y	
$5y$	
xy	
x^2	
y^2	
1	
5	
25	

Imagine these buildings are made by gluing Lab Gear blocks together. Find the total surface area of all the exposed faces of the arrangement, even the bottom of the building. Combine like terms in your answer and write it in the blank. (We will use SA for "surface area.")

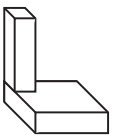
2. *SA* = _____



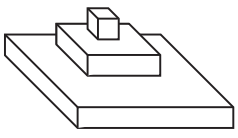
3. *SA* = _____



4. *SA* = _____

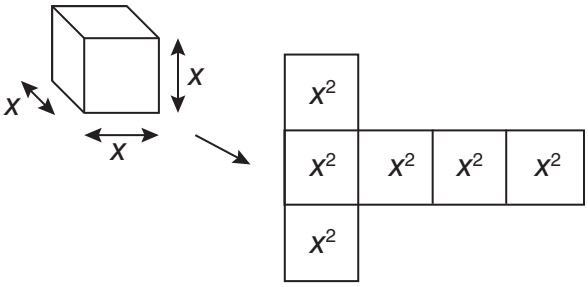


5. *SA* = _____



Surface Area Using 3-D Blocks

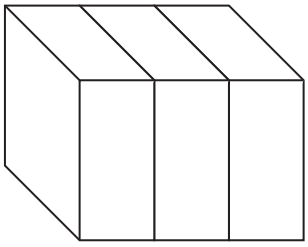
1. Take out one of each of the different 3-D Lab Gear blocks. Write the surface area of the block. Combine like terms to make your answer compact. Record your answer in the chart below. (The first one has been done as an example.)



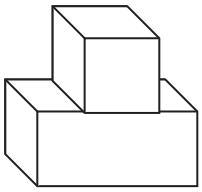
Lab Gear Block	Surface Area
x^3	$x^2 + x^2 + x^2 + x^2 + x^2 + x^2 = 6x^2$
y^3	
x^2y	
xy^2	

Imagine these buildings are made by gluing Lab Gear blocks together. Find the total surface area of all the exposed faces of the arrangement, even the bottom of the building. Combine like terms in your answer and write it in the blank.

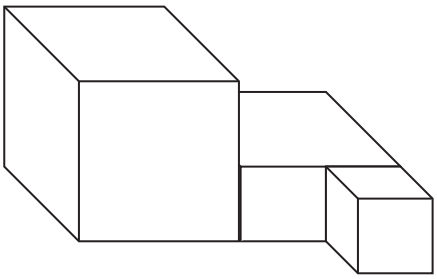
2. *SA* = _____



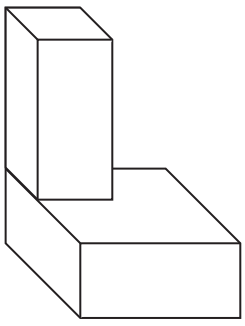
3. *SA* = _____



4. *SA* = _____



5. *SA* = _____



Surface Area Sequences

Look at each sequence of Lab Gear figures. Think about how it would continue, following the pattern. Then:

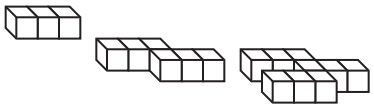
a. Sketch the next figure in the sequence.
b. Complete the table.
c. Describe the pattern in words.

1. a. 

c. _____
_____
_____

b.

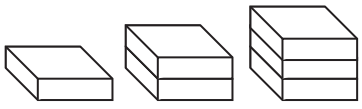
Figure #	Surface Area
1	10
2	
3	
4	
10	
100	

2. a. 

c. _____
_____
_____

b.

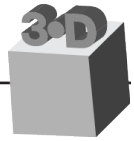
Figure #	Surface Area
1	14
2	
3	
4	
10	
100	

3. a. 

c. _____
_____
_____

b.

Figure #	Surface Area
1	$2x^2 + 4x$
2	
3	
4	
10	
100	



Make a Box Using 3-D Blocks

For each of the polynomials shown below:

a. Build the polynomial and arrange the blocks into a box.
b. Make a 3-D sketch of the box.
c. Complete the equation *volume = length · width · height*.

1. $x^2y + 4xy =$ _____

2. $x^3 + 5x^2 =$ _____

3. $xy^2 + x^2y + xy + x^2 =$ _____

4. $y^3 + x^2y + 2xy^2 =$ _____

5. $2x^2y + 2xy^2 =$ _____ = _____ = _____

   Find three different ways.

Multiplication Using the 3-D Blocks

The volume of a box can be found by multiplying the area of the base by the height. The base need not be horizontal. For each of the boxes described below:

a. Build the box with the Lab Gear.

b. Make a 3-D sketch of the box.

c. Write the volume of the box.

1. area of base = y^2
   height = $3x$
   volume = _____

2. area of base = $2xy$
   height = $x + 1$
   volume = _____

3. area of base = $3x + x^2$
   height = $3 + y$
   volume = _____

4. area of base = $xy + y^2$
   height = $y + x + 1$
   volume = _____

Use the Lab Gear to build boxes with the dimensions given below. Then write an equation in the form *length* · *width* · *height* = *volume*.

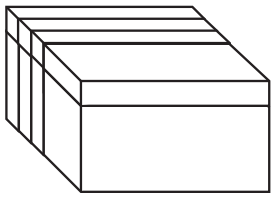
	Length	Width	Height	Volume
5.	$y + 1$	$y + 1$	y	
6.	$y + x + 2$	$x + y$	x	
7.	$x + 3$	$x + 2$	$x + 1$	

Three Factors Using the 3-D Blocks

For each problem:

a. Build a box with the Lab Gear.

b. Write or complete the equation *volume* = *length* · *width* · *height*.

1. 

Equation: _____

2. $y^3 + y^2 =$ _____

3. $x^3 + 8x^2 + 17x + 10 =$ _____

4. $xy^2 + 3xy + 2x =$ _____

5. $2x^2y + 2xy^2 + y^2 + 3xy + y =$ _____

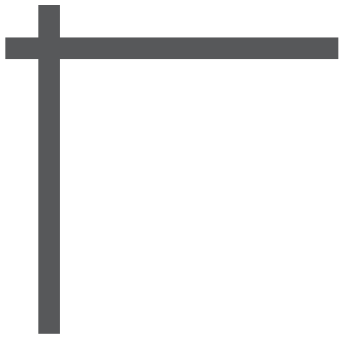
6. $x^3 + 4x^2 + 5x + 2 =$ _____

Dividing in the Corner Piece

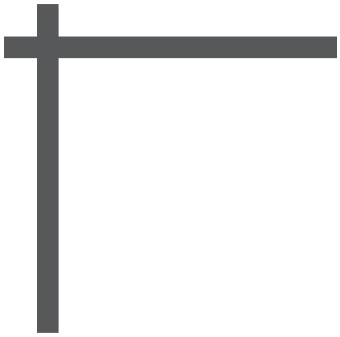
Use the corner piece and blocks to show each division. Sketch the blocks. Write the answer in the blank.

1. $\frac{6x^2 + 3x}{3x}$ = _____

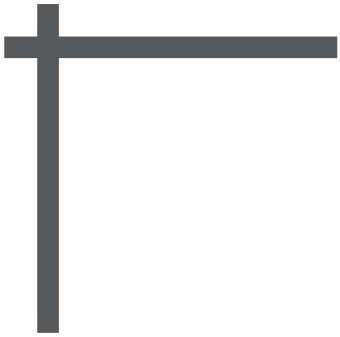
2. $\frac{4x^2 + 6x}{2x}$ = _____



3. $\frac{4x + 10}{2}$ = _____



4. $\frac{2x^2 + xy + 5x}{x}$ = _____



Find the answer to each division. Use the Lab Gear if you wish.

5. $\frac{7x + 14}{7}$ = _____

6. $\frac{2xy + y^2}{y}$ = _____

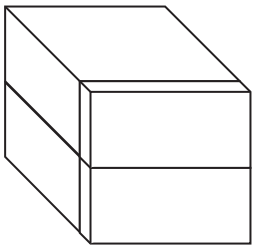
7. $\frac{3x^2 + 15x}{3x}$ = _____

8. $\frac{2x^2 + 2xy + 4x}{2x}$ = _____

Dividing in the Corner Piece Using the 3-D Blocks

The volume of a box can be found by multiplying the area of any rectangular face by the dimension perpendicular to that face.

1. The box shown below could be used to find the answers to three different division problems. Complete each of the equations.



$$\frac{2xy^2 + 2xy}{y} = \_\_\_\_\_$$

$$\frac{2xy^2 + 2xy}{y + 1} = \_\_\_\_\_$$

$$\frac{2xy^2 + 2xy}{2x} = \_\_\_\_\_$$

In problems 2 and 3, use the blocks to build a box that will help you find the answer to the division problem. Then complete two more division equations related to your box.

2. $\dfrac{y^3 + xy^2 + y^2 + xy}{y} = \_\_\_\_\_$

$\dfrac{y^3 + xy^2 + y^2 + xy}{\quad} = \_\_\_\_\_$

$\dfrac{y^3 + xy^2 + y^2 + xy}{\quad} = \_\_\_\_\_$

3. $\dfrac{x^3 + 4x^2 + 3x}{x + 1} = \_\_\_\_\_$

$\dfrac{x^3 + 4x^2 + 3x}{\quad} = \_\_\_\_\_$

$\dfrac{x^3 + 4x^2 + 3x}{\quad} = \_\_\_\_\_$

Patterns in Making a Square

Build each trinomial with the Lab Gear blocks. Arrange the blocks into a square. Complete the equation *area* = *side*^{2}. *Some of the trinomials cannot be made into a square.*

1. $x^2 + 8x + 16 =$ _____
2. $x^2 + 14x + 49 =$ _____
3. $x^2 + 10x + 25 =$ _____
4. $x^2 + 16x + 64 =$ _____
5. $x^2 + 4x + 16 =$ _____
6. $x^2 + 2x + 1 =$ _____
7. $x^2 + 4x + 4 =$ _____
8. $x^2 + 5x + 9 =$ _____
9. In each square, describe the pattern in which you arranged
   a. the *x*-blocks. _____
   _____
   b. the yellow blocks. _____
   _____
10. List the polynomials above that could not be made into a square. Explain what was wrong with each of them.

_____

_____

_____

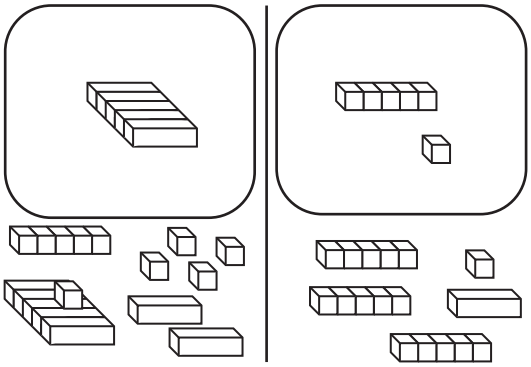
_____

_____

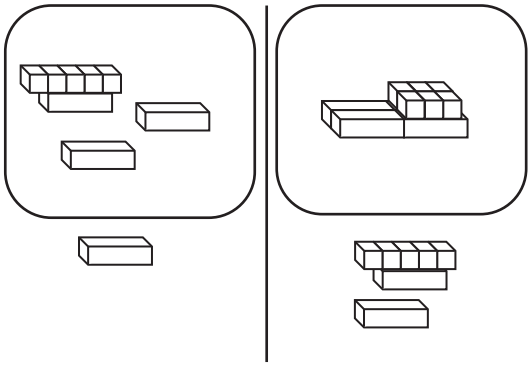
Which *x* Makes the Expressions Equal?

For each problem, simplify each side of the workmat and write the simplified expressions. Circle the greater expression. If it is impossible to tell which is greater, write the value of *x* that makes the two expressions equal.

1.



2.



For each of these problems, try to decide which of the two expressions is greater. Circle the greater expression. If it is impossible to tell which is greater, write the value(s) of *x* that make the two expressions equal.

3. x $2x + 3$

4. $4x$ $4x + 5$

5. $3x + 4$ $5x + 30$

6. $6x$ $7x^2 + 6x - 7$

7. $8x + 9$ $-x^2 + 5x + 9$

8. $2(x - 2) + 5$ $6 + 5x + 1 - 3(x - 1)$

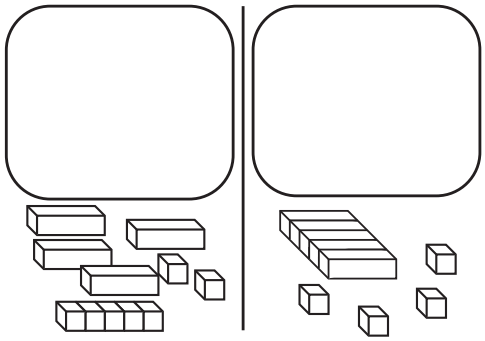
Solving Linear Equations

Use the Lab Gear to solve each of the equations shown below.

a. Write the equation shown.

b. Record some of the equations you encounter along the way, as well as the final answer.

1.



a. _____

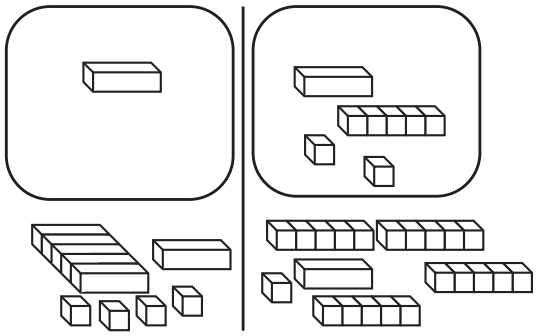
b. _____

_____

_____

_____

2.



a. _____

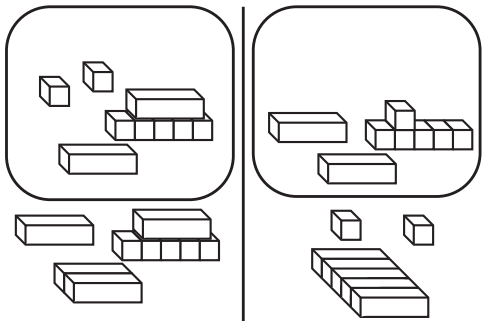
b. _____

_____

_____

_____

3.



a. _____

b. _____

_____

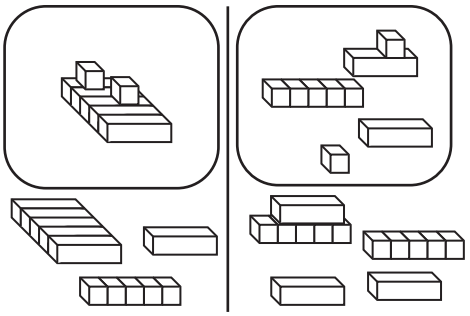
_____

_____

Solving Two Ways

Work with a partner. One of you should solve an equation with the Lab Gear, and the other on paper. If you do the work correctly, you should get the same answer! Switch solution methods with your partner each time you begin a new problem.

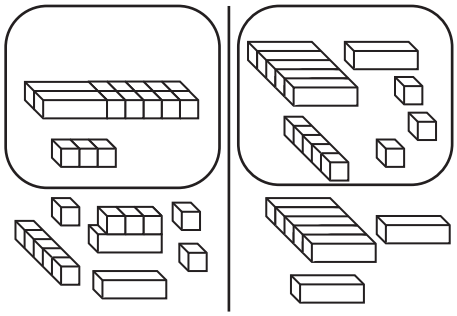
1.



_____
(original equation)

_____
(solution)

2.



_____
(original equation)

_____
(solution)

3. $2x - 11 - (3x + 1) = 5x - (4x + 2)$ Solution: _____

4. $25 - 3x - (x + 2) = 11x + 15 - (x - 1)$ Solution: _____

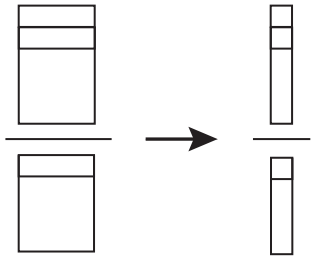
5. $2(x - 3) - (5x + 1) = 2(2 - x) - (3x - 1)$ Solution: _____

6. $x + 5(x - 1) + 1 - (x^2 + 1) = x + 10 - (x^2 + 5)$ Solution: _____

Simplifying Fractions

1. Name the fraction that is being simplified in this figure.

2. Name the simplified fraction.



Simplify each fraction. *One fraction cannot be simplified.*

3. $\dfrac{2x+6}{2y+10} =$ _____

4. $\dfrac{xy+y}{y^2+y} =$ _____

5. $\dfrac{x^2+3x+2}{x^2+4x+3} =$ _____

6. $\dfrac{x^2+4x}{x^2+5x+4} =$ _____

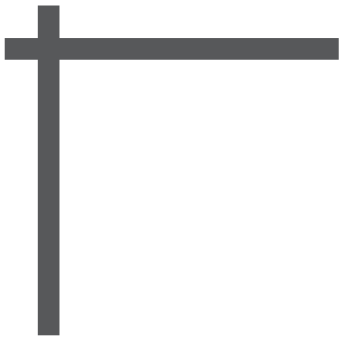
7. $\dfrac{x^2+10x+25}{x^2+5x} =$ _____

8. $\dfrac{x^2+2x+1}{x^2+2x} =$ _____

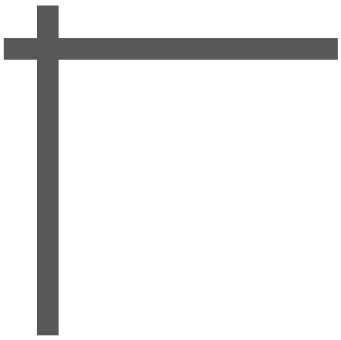
Minus in the Corner Piece

For problems 1–4, multiply in the corner piece, sketch the end result, and combine like terms. Check your answers by multiplying without the Lab Gear.

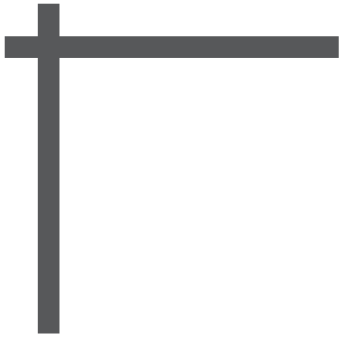
1. $(y + 2)(y + 4) =$ _____



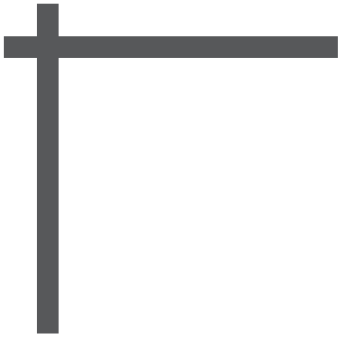
2. $(y - 2)(y + 4) =$ _____



3. $(y + 2)(y - 4) =$ _____



4. $(y - 2)(y - 4) =$ _____



For problems 5–10, multiply and combine like terms.

5. $(2x + 5)^2 =$ _____

6. $(2x + 5)(2x - 5) =$ _____

7. $(2x - 5)^2 =$ _____

8. $(2x + 1)^2 =$ _____

9. $(2x + 1)(2x - 1) =$ _____

10. $(2x - 1)^2 =$ _____

The Zero Product Principle

Suppose a and b are numbers. Indicate whether these statements are true or false by writing T or F in the blank. Explain each answer.

1. If $a = 0$ and $b = 0$, then $ab = 0$. ____
2. If $a \neq 0$ and $b = 0$, then $ab = 0$. ____
3. If $a = 0$ and $b \neq 0$, then $ab = 0$. ____
4. If $a \neq 0$ and $b \neq 0$, then $ab = 0$. ____
5. Copy statement 4, changing one symbol to make it true. (Do not change it to any of the statements 1–3.)

_____

6. If you know that $ab = 0$, what can you conclude about a and b?

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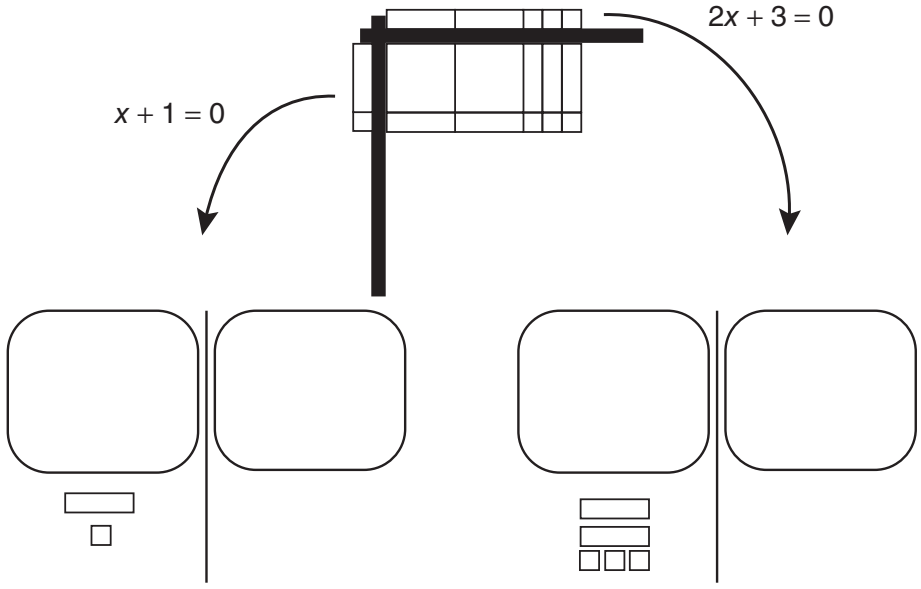
The equation $(x + 1)(2x + 3) = 0$ can be solved by using the Lab Gear and the zero product principle. The solutions are $x = -1$ and $x = -1.5$.



Solve each equation. Write the solution in the blank.

7. $(3x + 1)(x + 5) = 0$ _____ 8. $(5x + 10)(2x + 4) = 0$ _____

9. $(x + 4)(3x + 5) = 0$ _____ 10. $2x^2 + 8x = 0$ _____

Solving by Factoring

Solve each equation, using factoring and the zero product principle. Use the Lab Gear or multiplication table format to factor. Show your work. *One cannot be factored.*

1. $x^2 + 7x + 12 = 0$

2. $x^2 + 5x + 6 = 0$

3. $x^2 + 11x + 10 = 0$

4. $x^2 + 8x + 15 = 0$

5. $x^2 + 13x = 0$

6. $x^2 + 2x + 4 = 0$

7. $x^2 + 5x + 4 = 0$

8. $4x^2 + 4x + 1 = 0$

9. $4x^2 + 10x = 0$

10. $3x^2 + 4x + 1 = 0$

11. $9x^2 + 21x + 6 = 0$

12. $2x^2 + 7x + 5 = 0$

Solving by Equal Squares

Solve the following equations using the equal squares method. Show your work. One problem is impossible. (The solutions are not necessarily integers.)

1. $x^2 = 64$

2. $x^2 + 6x + 9 = 0$

3. $9x^2 = 36$

4. $x^2 + 14x + 49 = -16$

5 $x^2 + 10x + 25 = 1$

6. $4x^2 + 12x + 9 = 25$

7. $4x^2 + 4x + 1 = 36$

8. $x^2 + 8x + 16 = 4$

9. Explain why some problems had only one solution, or no solution.

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_____

Solving by Completing the Square

Solve each equation by first completing the square and then using the equal squares method. Show your work. *One problem is impossible*. (The solutions are not necessarily integers.)

1. $x^2 + 4x = 5$

2. $x^2 + 6x = 16$

3. $x^2 + 10x = -24$

4. $x^2 + 8x + 20 = 8$

5. $x^2 + 4x + 9 = 2$

6. $4x^2 + 8x - 5 = 0$

7. $2x^2 + 6x = 2 - 6x$

8. $x^2 + 12x + 26 = -3$