

a

CCSSM

Two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence



Congruence and Similarity pp. 1-8



Consequences for proof in geometry

Different postulates

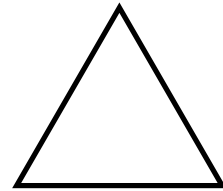
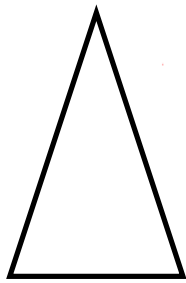
Some new definitions

Some new strategies for proof

Isosceles Δ : has line symmetry

Equilateral Δ : has rot. sym.

New definitions: special triangles



New definitions: special quadrilaterals

Parallelogram

rotational sym.

Rectangle

reflection through
midpoints of sides

Rhombus

both diag are lines of sym.

Square

4-fold rot.



Kite

one diag. is line of sym.

Trapezoid

one side is a dilation of the other

Isosceles Trapezoid

one line of sym through
midpts

Definition: An *isometry* is a transformation of the plane that preserves distance.

Definition: Two figures are *congruent* if one can be superposed on the other by a sequence of isometries.

Definition: A *reflection* in a line b maps any point on b to itself, and any other point P to a point P' so that b is the perpendicular bisector of PP' .

Proposed postulate:

- Reflections preserve distances and angles

"Construction postulates":

- Two lines meet in at most one point.
- A circle and a line meet in at most two points.
- Two circles meet in at most two points.

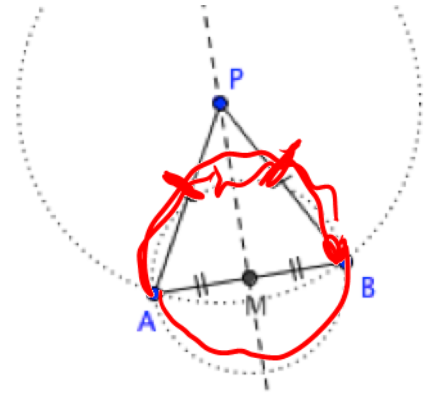
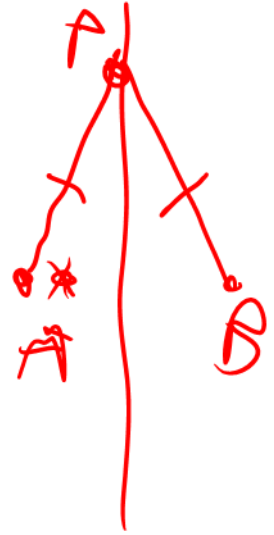
There is a reflection that maps any given point P into any given point Q.



Perpendicular bisector theorems

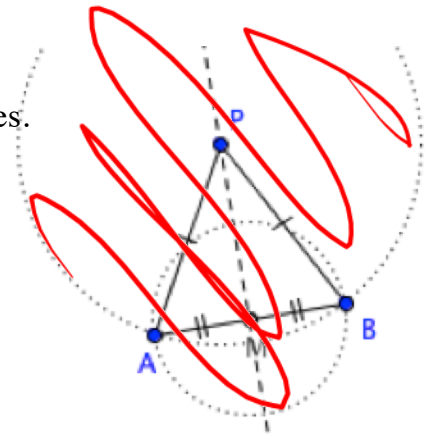
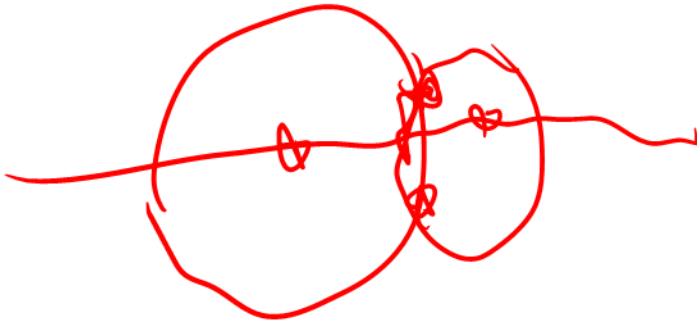
Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

If a point is equidistant from the endpoints of the segment, it is on the perpendicular bisector of the segment.

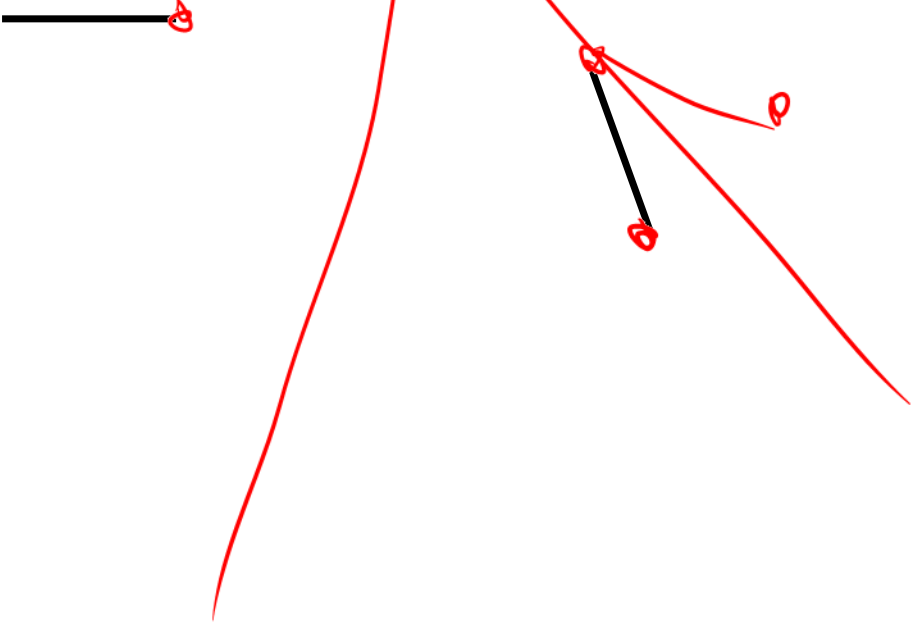


(A point is equidistant from the endpoints of a segment if and only if it lies in the perpendicular bisector of the segment.)

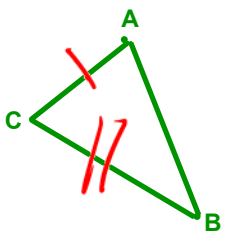
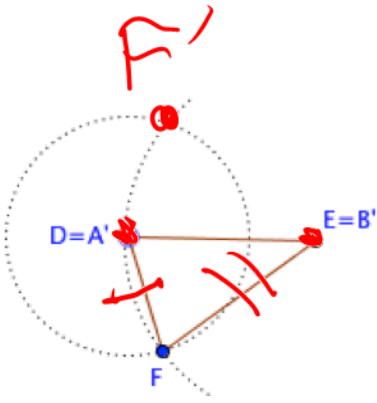
Corollary: If two circles intersect in two points, those points are reflections of each other in the line joining the centers of the circles.



Equal-length segments are congruent

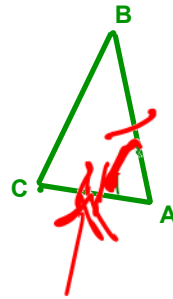


SSS



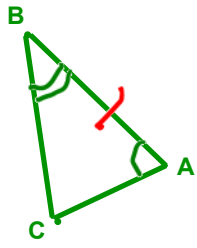
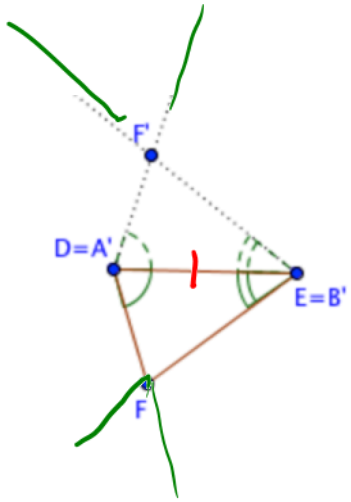
Handwritten marks in green and red.

SAS



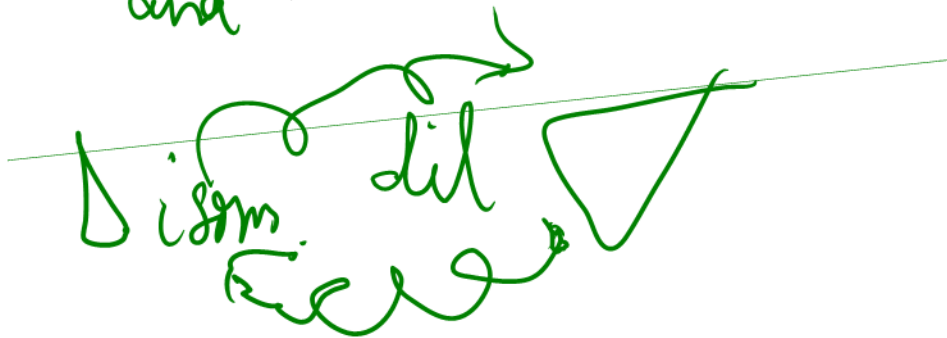


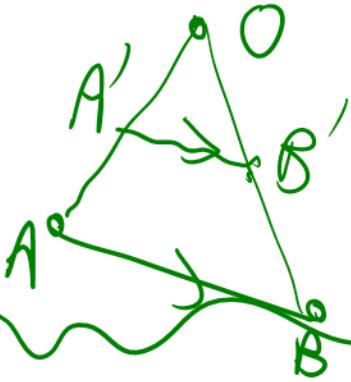
ASA



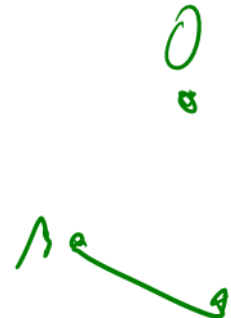
Definition: A dilation with center O and scaling factor r maps O to itself and any other point P to P' so that the directed segment $OP' = r \cdot OP$.

Definition: Two figures are *similar* if one can be superposed on the other by a sequence of isometries followed by a dilation.
and

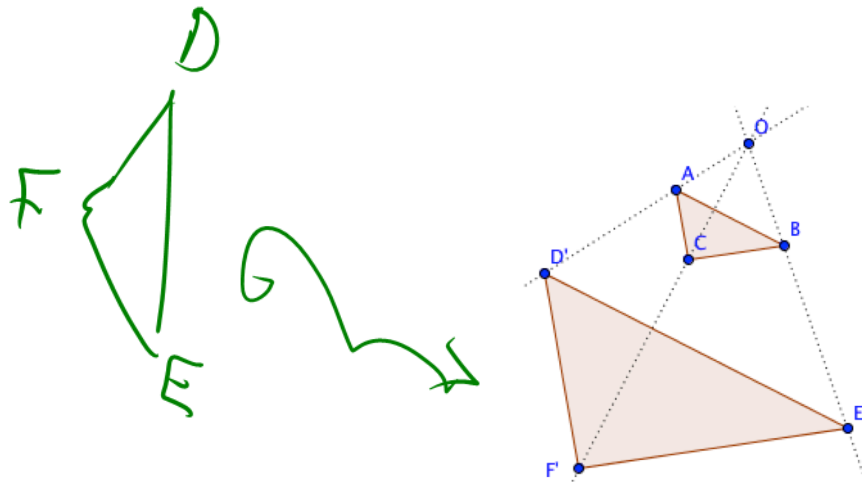




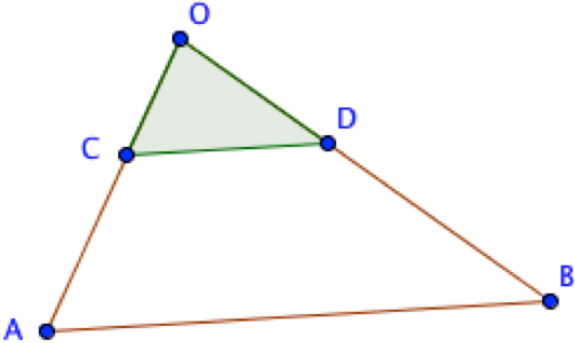
Assumption: If O , A , and B are not collinear, the image $A'B'$ of the segment AB under a dilation with center O and scaling factor r is parallel to AB , with length $r \cdot AB$.



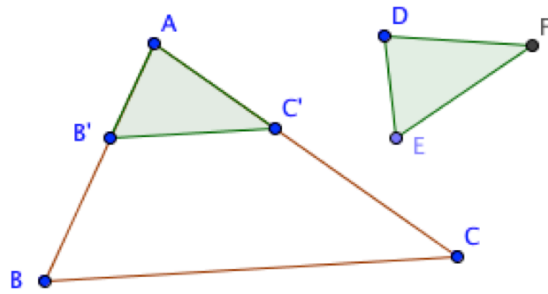
Similar triangles have angles with equal measure and proportional sides.



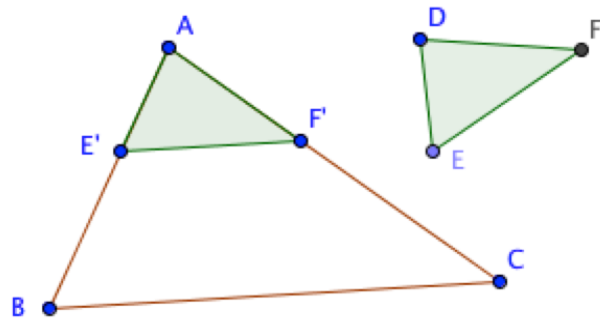
If two segments are parallel and unequal, one is the image of the other under a dilation.



SSS



SAS



AA

