Hands-On Geometry With Henri Picciotto

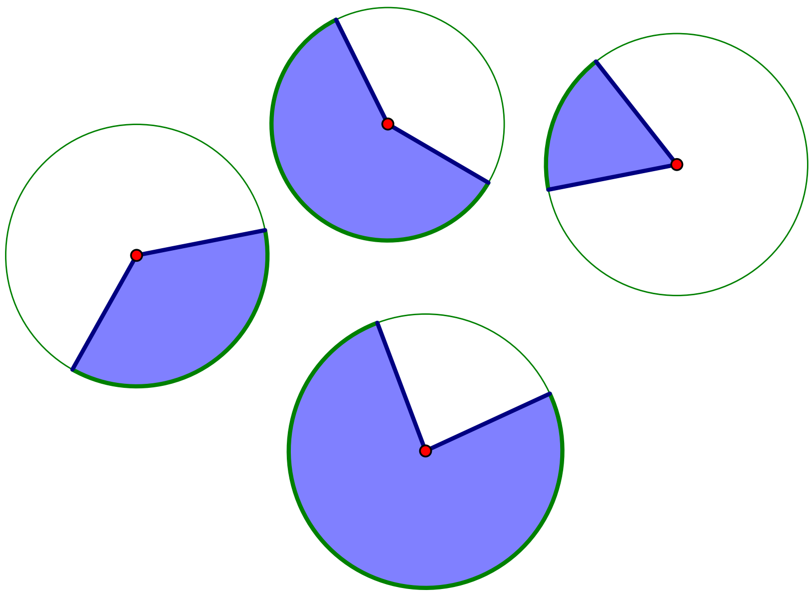
Head-Royce School

June 26th and 27th, 2017

Guest Presenter: Rachel Chou

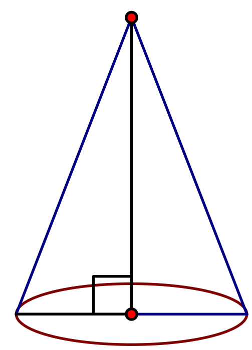
Surface Area of Cones

"You can build a cone from a sector of a circle, and some tape! A sector of a circle is a region of a circle bounded by two radii (like a piece of pizza.) Make a cut along a radius, and then make another cut! You have a sector. Sectors of circles are depicted below!"



A cone is shown below. A cone has 3 main measurements that you could take with a ruler:

(1) A *radius*.
(2) A *height*.
(3) A *slant height*.



1. Label each of the above terms on the cone to the right.

2. You have been given a circle with two radii drawn. These two radii split the circle into a smaller sector and a larger sector. Cut out the larger sector, and use tape to form the ***lateral surface*** of a cone.

3. Your cone happens to have a radius of 6.4cm and a ***slant height*** of 8cm. Given these facts, find:

   (a) The actual height of the cone.
   (b) The lateral surface area of the cone. (Leave your answer in terms of π.)
   (c) The angle of the sector that you used to build the cone?

4. If you had a cone of radius = 5cm and slant height = 12cm,

   (a) What would be the height of the cone?
   (b) The ***lateral surface area*** of the cone? (Leave your answer in terms of π.)

5. For a cone of radius: r, and slant height l, find:
   (a) The actual height.
   (b) The lateral surface area. (Leave your answer in terms of π.)

6. A semi-circle of radius 6cm forms the lateral surface of a cone. Find this cone's ***total surface area*** (including the base!).

7. A cone has a lateral surface area of 24π and a radius of 6. Find:
   (a) The slant height.
   (b) The height.

Introduction to Triangle Congruence

INTRODUCTION/INSTRUCTIONS: Today we explore what might be required to **guarantee** two triangles congruent. Work on the following exercises.

1. It is known that $\Delta MTH \cong \Delta KWL$. The previous statement conveys that two triangles are congruent. Furthermore, the order in which the vertices are written is important. For example, from the previous statement, I might deduce that $\Delta HMT \cong \Delta LKW$, but I cannot deduce that $\Delta MHT \cong \Delta WKL$.

   (a) Fill in the blank: $\Delta MHT \cong$ Δ ______________________

   (b) Fill in the blank: $\Delta TMH \cong$ Δ ______________________

   (c) Name three pairs of congruent angles.

   (d) Name three sets of congruent sides.

In the following problems, you may use a ruler, a protractor, and possibly a compass. In every case, draw the triangle described and then comment on whether or not you think your triangle is congruent to everyone else's in the class.

2. ΔRST with the following characteristics. $m\angle R = 35°$, $m\angle S = 65°$, and $m\angle T = 80°$.
3. ΔKLM with the following characteristics. $KL = 6$cm, $LM = 8$cm.
4. ΔABC with the following characteristics. $AB = 6$cm, $BC = 8$cm, and $m\angle B = 40°$.
5. ΔDEF with the following characteristics: $m\angle D = 70°$ and $m\angle E = 60°$.
6. ΔNPR with the following characteristics: $NR = 7.5$cm, $m\angle N = 70°$, and $m\angle R = 60°$.
7. ΔGHI with the following characteristics: $GH = 7.5$cm, $m\angle G = 78°$, and $m\angle H = 108°$.
8. ΔXYZ with the following characteristics: $XY = 6$cm, $m\angle Y = 54°$, and $m\angle Z = 60°$.[1]
9. ΔABC with the following characteristics: $AB = 6$cm, $BC = 9$cm, $AC = 12$cm.

[1] Hint: Find $m\angle X$ first.

10. Look over your work on this worksheet. Then answer the following questions:
   (a) When two triangles are congruent, we may write ________ statements of segment congruence and

   __________ statements of angle congruence for a total of __________ congruence statements.

   (b) How many statements of congruence are often required to **guarantee** triangle congruence?

11. What minimal sets of information might we use to guarantee triangle congruence?

12. Draw ΔABC with the following characteristics. $AB = 8$cm, $m\angle B = 40°$, and $AC = 6$cm. Do you think that your ΔABC is congruent to everyone else's?

Re-visiting SSA

1. For each of the following stated criteria, say whether or not (Yes or No) a triangle with this criteria could exist. Then comment on whether or not I have described a **unique** triangle. (Yes or No).

Criteria	**Exists?**	**Unique?**
$\Delta ABC;\ AB = 10;\ m\measuredangle A = 70°;\ BC = 11.$		
$\Delta DEF;\ DE = 10;\ m\measuredangle D = 104°;\ EF = 9.$		
$\Delta GHI;\ GH = 10;\ m\measuredangle G = 40°;\ HI = 9.$		
$\Delta JKL;\ JK = 10;\ m\measuredangle J = 40°;\ KL = 2.$		

2. Consider ΔABC with $AB = 10$ and $m\measuredangle A = 35°$.
   (a) For what lengths BC, will we be able to make 2 different triangles?
   (b) For what lengths BC, will we be able to make 1 unique triangle?
   (c) For what lengths BC, will no triangle be possible?

SSS? Does SSSS work? What about SSSSS?

In your table groups, you have been given some straws on a string. Depending on the number of straws, if you tie the string ends in a knot, you might build a triangle, a quadrilateral, or a pentagon. Tie all the ends in a nice knot to build your shapes. Then answer the questions below.

1. If everyone in a class draws a triangle with the following side lengths: 5cm, 6cm, 7cm, will everyone's triangle be congruent?

2. If everyone in a class draws a triangle with the following side lengths: 5cm, 9cm, 17cm, will everyone's triangle be congruent?

3. Two quadrilaterals are ***equilateral*** (equal sides) with a perimeter of 20cm. Are they definitely congruent?

4. Two pentagons have the following side lengths in the following order: 5cm, 5cm, 6cm, 8cm, 10cm.

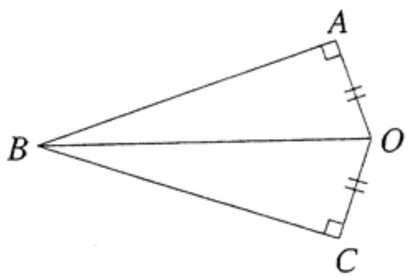
   They are __________________________ congruent. Fill in the blank with one of the words: *always*, *sometimes*, or *never*.

5. How many possible quadrilaterals can be drawn with the following side lengths (in the following order):
   (a) 5cm, 6cm 20cm, 4cm, 4cm
   (b) 5cm, 6cm 10cm, 4cm, 8cm

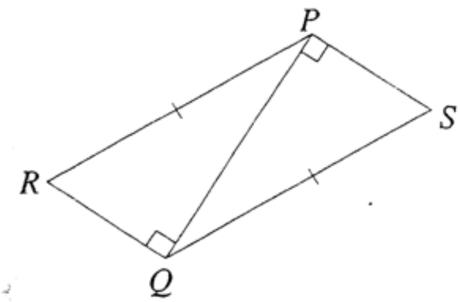
One More Congruence Method

In your groups, try to decide if you can declare any of the triangles shown below congruent.

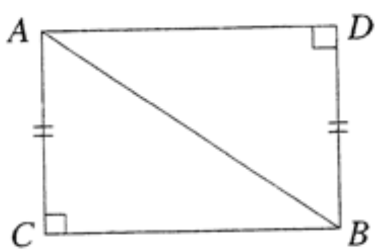
(a)



(b)

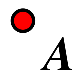


(c)



Does this triangle exist? Is it unique? Hmmm…..

1. What would you create if you drew in all of the points 5cm away from point A shown below?



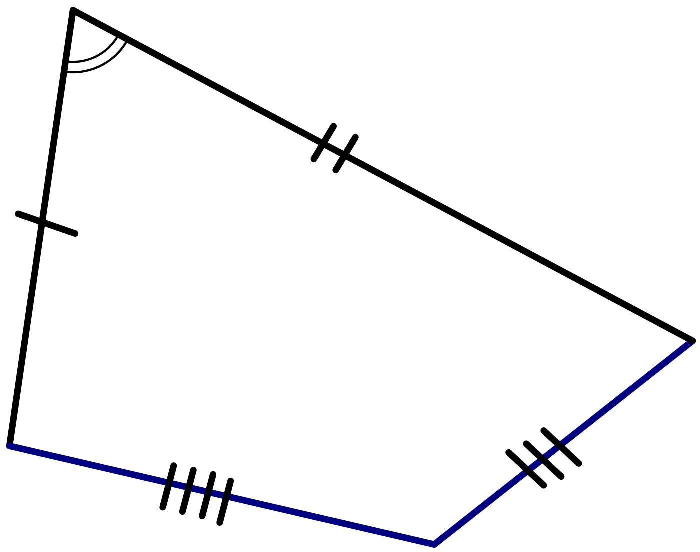
2. Mr. Picciotto asks his students to all draw ΔABC which meets the following criteria:
   $m\measuredangle A = 35°$; $\overline{BC} = 8cm$; $\overline{AB} = 10cm$
   (a) If you were to draw this triangle, which side might you want to start with: $\overline{AB}$ or $\overline{BC}$? Why?
   (b) Draw a triangle which meets this criteria.
   (c) How many possible triangles **can** you draw which meet this criteria?

3. For each of the following stated criteria, say whether or not (Yes or No) a triangle with this criteria could exist. Then comment on whether or not I have described a **unique** triangle. (Yes or No).

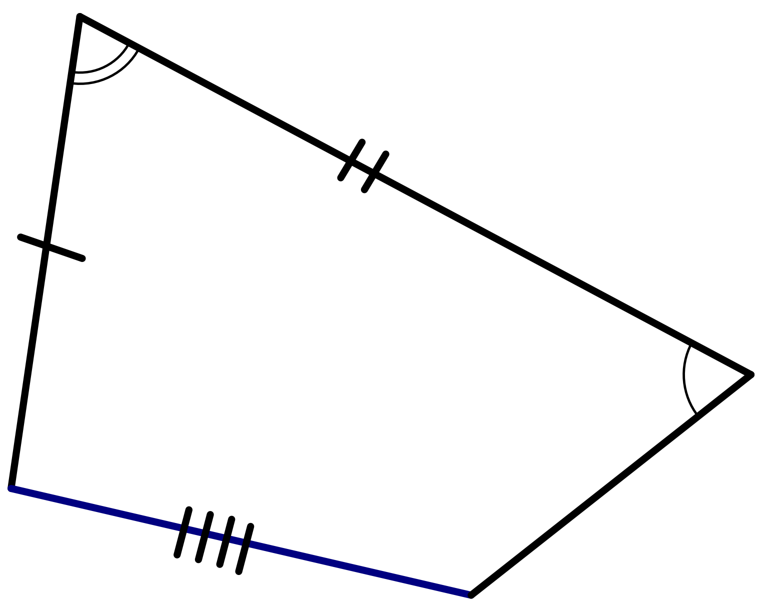
Criteria	**Exists?**	**Unique?**
ΔABC; $m\measuredangle A = 54°$; $m\measuredangle B = 120°$.		
ΔDEF; $m\measuredangle D = 54°$; $m\measuredangle E = 128°$.		
ΔGHI; $m\measuredangle G = 54°$; $m\measuredangle H = 50°$; $GH = 10$.		
ΔMNO; $MN = 10$; $NO = 8$; $m\measuredangle N = 100°$		
ΔPQR; $PQ = 10$; $QR = 8$; $PR = 20$		
ΔSTU; $ST = 10$; $TU = 8$; $SU = 12$		
ΔAYZ; $AY = 10$; $YZ = 8$; $AZ = 12$; $m\angle A = 100°$		
ΔBCD; $BC = 10$; $CD = 8$; $m\angle B = 100°$		
ΔEFG; $EF = 10$; $FG = 8$; $m\angle F = 30°$		
ΔHIJ; $HI = 10$; $IJ = 12$; $m\angle H = 30°$		

Quadrilateral Congruence

1. Does the information SASSS guarantee quadrilateral congruence? Why or why not?



2. Does the information SSASA guarantee quadrilateral congruence? Why or why not?



Existence and Uniqueness

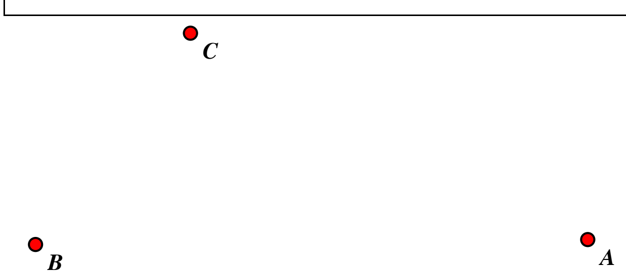
We have discussed the importance of making clear instructions or descriptions. Two ways to help us assess if our descriptions are clear is to ask:

(1) Did I describe something ***unique***?
(2) Did I describe something which actually ***exists***.

Each of the following descriptions or instructions may break either the **uniqueness rule** or the **existence rule**. For each example,

(1) **Decide whether or not the instruction is "clear**."
(2) If it is not clear, **state which rule is being broken** (existence or uniqueness), and be prepared to elaborate on your choice, if asked!

Statement:	**Clear?**	**Rule?**
1. **"**Please go find Henry."		
2. "Please go find Henri Picciotto." (Assume there is only one Henri Picciotto.)		
3. "Please go find Rachel Chou with her hair in a ponytail."		
4. Draw a line trough point B.		
5. Draw a line through points A and B.		
6. Draw a line through points A, B, and C.		
7. Draw a line through point B perpendicular to $\overline{AB}$.		
8. Draw a line through the midpoint of $\overline{AB}$, perpendicular to $\overline{AC}$.		
9. Draw a line through the midpoint of $\overline{AB}$, and also point C, and also perpendicular to $\overline{AB}$.		
10. Draw the angle formed by $\overrightarrow{AC}$ and $\overrightarrow{AB}$.		
11. Draw a 70° angle formed by $\overrightarrow{AC}$ and $\overrightarrow{AB}$.		



Name: ________________________ Date: _________________________

Shrinky Dink® Fun!

BACKGROUND: You are probably familiar with Shrinky Dinks—a craft project that involves producing a design on a plastic sheet and heating it. During the heating process, the plastic shrinks, producing a reduced (but roughly proportional) version of the original designs.

INSTRUCTIONS:

1. Use a pencil and lightly outline your drawing on your Shrinky Dink® paper. Before you begin your drawing, be certain of what you want to draw, as the paper is somewhat expensive. Tracing a picture out of a magazine or book, or finding something online to trace is a great idea. Do not pick any picture or design that is long and skinny, for such designs will curl when they are heated. Avoid a shape that is a simple square or rectangle. Use a hole-punch to make a hole that you can use to hang your design on the bulletin board.

2. Use colored pencils or permanent markers to color your picture.

3. Cut your design out carefully.

4. With a **metric** ruler, measure the width of your cutout at its **widest** point and at its **tallest** point. Record your measurements here:

   Original **widest** point: __________________cm. Original **tallest** point: ____________cm.

5. Use graph paper to approximate the area of your cutout in square units. Each square on the graph paper equals 1 square unit. By tracing your cutout onto graph paper, you should be able to estimate the approximate area of your cutout. Attach the graph paper to this handout and show how you calculated the area of your design.

   My cutout is approximately ___________________ graph paper square units.

6. Turn in your cutout to me. I will bake it for you and return it to you at the next class period.

7. When you get your cutout back, measure the height of the Shrinky Dink at its **tallest** point at at its widest point

   New **tallest** point: _________cm. New **widest** point: _________cm.

8. Compare your measurements in #8 to the measurements that you took in #4 and #5. How do they compare? What is the ratio of the old height to the new height? What is the ratio of the old width to the new width?

   $\frac{\textit{Old height}}{\textit{New height}} =$ $\frac{\textit{Old width}}{\textit{New width}} =$

9. Use graph paper to approximate the area of your Shrinky Dink. Sketch the design next to the original and show the method you use to calculate the new area.

   My Shrinky Dink is approximately ____________________ graph paper square units.

10. Compare your second area measurement to your measurement in #5. How do they compare? What is the ratio of the old area to the new area?

$$\frac{\textit{Old area}}{\textit{New area}} =$$

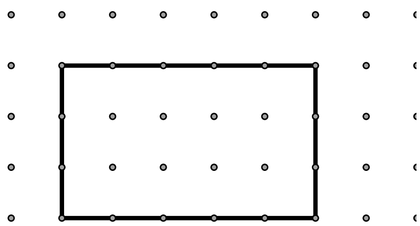
11. Are your ratios in #8 and #10 the same? Are they different? How **should** they relate?

12. Your Shrinky Dink is much thicker now that it has been baked. How much thicker do you think it is? Twice as thick? Three times as thick? 5 times as thick? Explain your reasoning.

Area of Triangles

1. Find the area:

(a)



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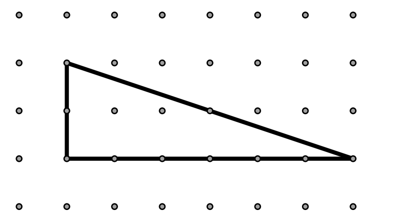

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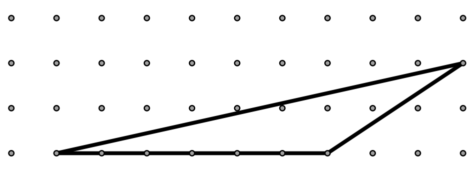

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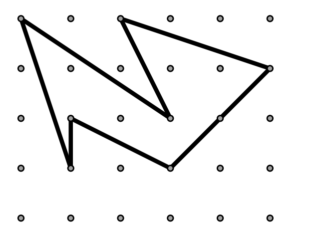
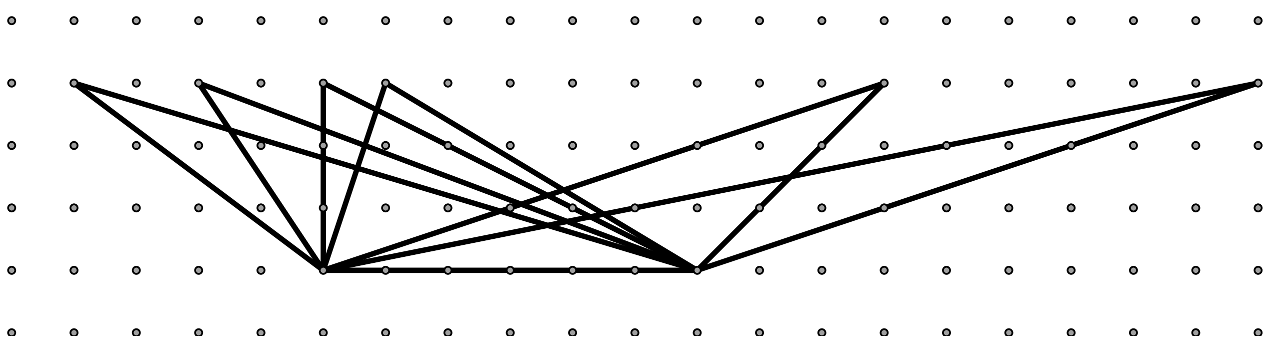


2. What happens to the area of a triangle as one vertex moves in a direction parallel to the side opposite this vertex?



3. Classic challenge problem: 7 people want to share an 7inch by 7inch cake. The cake is frosted on the top and on the sides. Everyone wants the same amount of cake and the same amount of frosting. How do you cut the cake to achieve this goal?