Complex Numbers

A Graphical Approach

$$\sqrt{-1} = i$$

What does this mean?

a. $x + 2 = 9$	b. $x + 9 = 2$
c. $2x = 6$	d. $6x = 2$
e. $x^2 = 9$	f. $x^2 = 10$
g. $x^2 = -9$	

Positive whole numbers : a, c, e

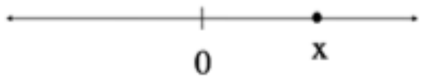
Integers : a, c, e, b

Rational numbers : a, c, e, b, d

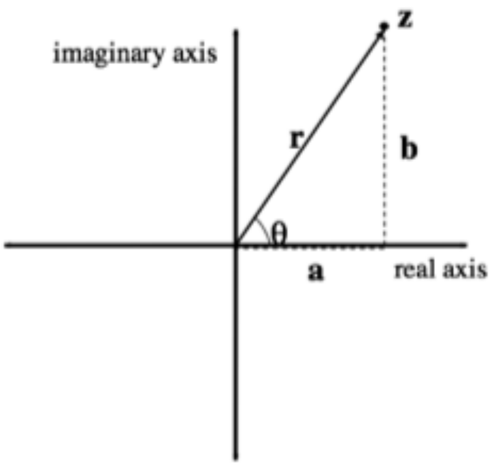
Real numbers : a, c, e, b, d, f

Complex numbers: adds g to the list

The Real Number Line



The Complex Number Plane



Polar form vs. rectangular form

$a = r \cos \theta$

$b = r \sin \theta$

$z=(a,b) \longrightarrow z=a+bi$

$z=(r, \theta)$

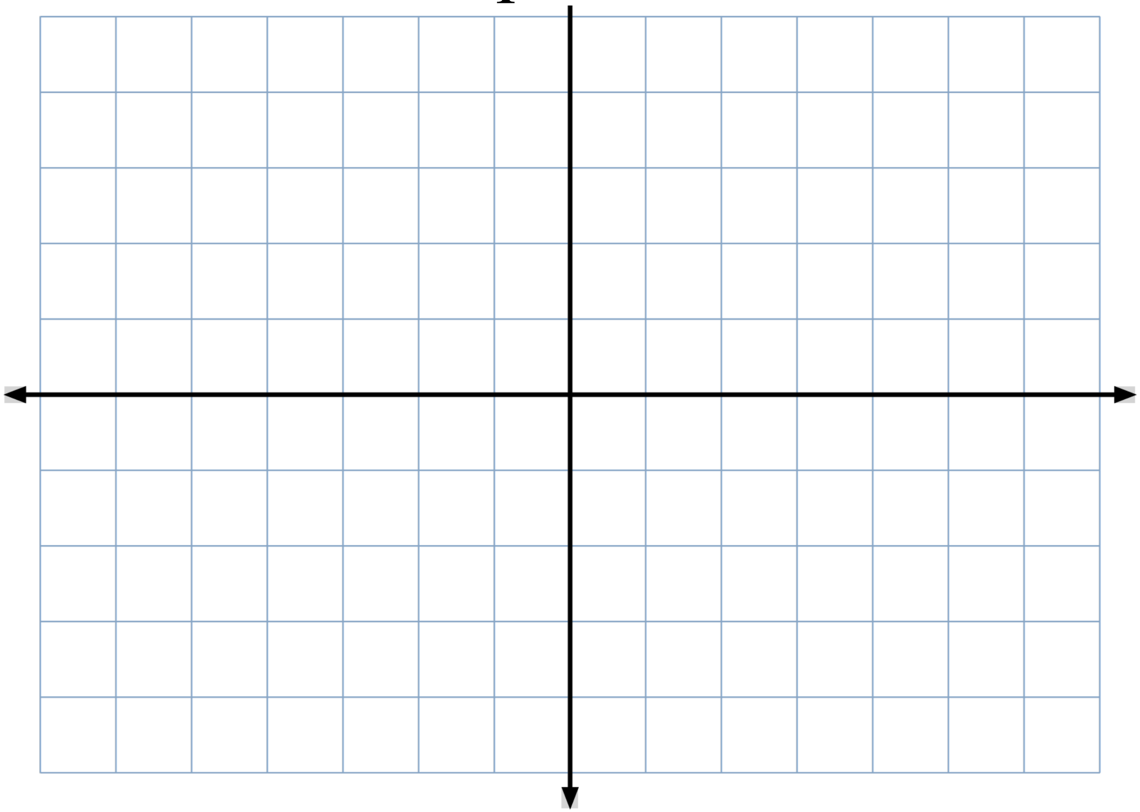
$z=(r \cos \theta) + (r \sin \theta)\, i$

Addition / subtraction are straightforward

a + bi form
visually: works like vectors algebraically:

add or subtract the real parts,
add or subtract the imaginary parts

Addition of complex numbers



Multiplication of Complex Numbers

Given $z_1 = (r_1, \theta_1)$ and $z_2 = (r_2, \theta_2)$

$$(r_1, \theta_1) \cdot (r_2, \theta_2) = (r_1 \cdot r_2, \theta_1 + \theta_2)$$

An Example:

$$(2, 30°) \cdot (3, 90°) = (6, 120°)$$

Does that work for real numbers?

$(2,0°)\cdot(3,0°)$

$(2,0°)\cdot(3,180°)$

$(2,180°)\cdot(3,180°)$

- One (1,0°), remains the multiplicative identity.

$$(1,0^\circ)\cdot(r,\Theta)=(r,\Theta)$$

- Reciprocals are well-defined.

$$z\cdot\frac{1}{z}=1,\ z\neq 0$$
$$(r,\theta)\cdot\left(\frac{1}{r},-\theta\right)=(1,0^\circ),\ r\neq 0$$

- So division works.

$$\frac{z_1}{z_2}=z_1\cdot\frac{1}{z_2}$$
$$\frac{(r_1,\theta_1)}{(r_2,\theta_2)}=(r_1,\theta_1)\cdot\left(\frac{1}{r_2},-\theta_2\right)=\left(\frac{r_1}{r_2},\theta_1-\theta_2\right)$$

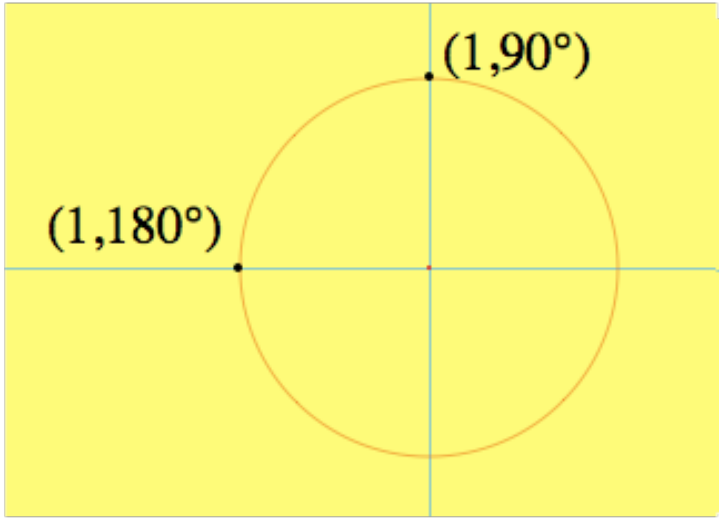
Powering

$(1,45°)^1 = (1,45°)$
$(1,45°)^2 = (1,90°)$
$(1,45°)^3 = (1,135°)$
$(1,45°)^4 = (1,180°)$
$(1,45°)^5 = (1,225°)$
$(1,45°)^6 = (1,270°)$
$(1,45°)^7 = (1,315°)$
$(1,45°)^8 = (1,360°)$

$$(1,90°)^2 = ?$$

$$(1,90°)^2 = (1,180°)$$

$$(1, 90°)^2 = -1$$



$$\sqrt{-1} = (1,90°)$$

$$\boxed{\sqrt{-1} = i}$$

Complex Numbers Games

Computing Any Isometry Using Complex Numbers

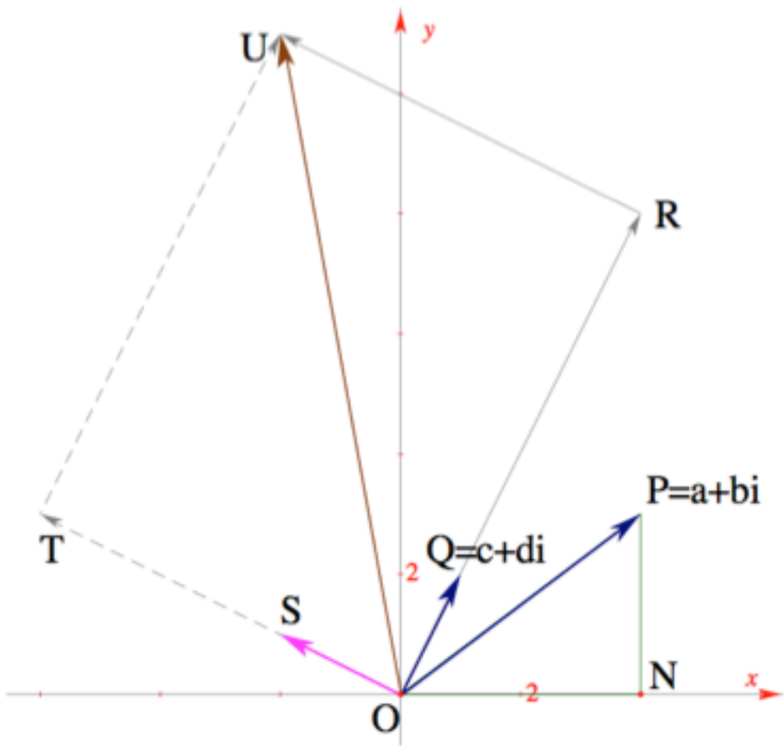
p. 9

Slope of perpendicular lines

p. 5

Proof that this makes sense p. 7 (Practice with numbers)

Actual proof p. 8


U
y
R
P=a+bi
T
Q=c+di
2
S
N
x
2
O

Looking back: this completes a quest that started in kindergarten

Looking ahead: this provides an approach to some trig identities.

(sin and cos of a sum, double angle formulas)

It also prepares students for geometric transformation matrices.