

Given: see figure

$AB$  is congruent to  $DE$ , so  $DE$  is the image of  $AB$  in a series of reflections.

So  $A'=D$ ,  $B'=E$ . Where is  $C'$ ?

Because distances are preserved we have  $A'C'=AC=DF$ , so  $C'$  must be on the circle centered at  $D$ , with radius  $DF$ .

Because angles are preserved,  $C'$  is either on ray  $DF$ , or on ray  $DF$ 's reflection across  $DE$ .

There are two possibilities, depending on which of the two rays  $C'$  is on.

If  $C'=F$ , we are done:  $\triangle A'B'C'$  is  $\triangle DEF$ , so we have superposed  $\triangle ABC$  onto  $\triangle DEF$  with a series of reflections, and thus they are congruent.

If  $C' \neq F$ , reflect  $\triangle A'B'C'$  in  $DE$ .  $A''=A'=D$ , and  $B''=B'=E$ . Because angles and distances are preserved,  $C''$  is at the intersection of the circle centered at  $D$  with radius  $DF$ , and the ray that makes an angle equal to angle  $A$  with side  $DE$ .

So  $C''=F$ . Therefore  $\triangle A'B'C''$  is  $\triangle DEF$ , and we have superposed  $\triangle ABC$  onto  $\triangle DEF$  with a series of reflections, and thus they are congruent.